Exercise 2.2.9

(Backwards again, now from solutions to equations) Find an equation $\dot{x} = f(x)$ whose solutions $x(t)$ look like those shown in Figure 2.

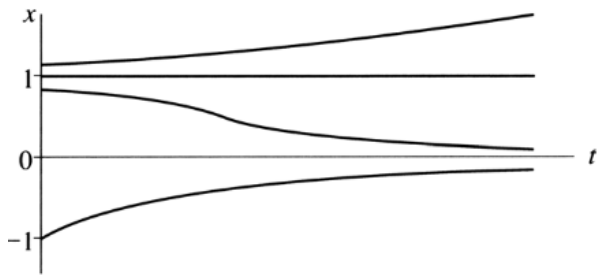

Figure 2

Solution

Observe that there are two fixed points, $x^* = 0$ and $x^* = 1$, which are locally stable and locally unstable, respectively. For an initial condition $x(0) > 1$, the slope is positive; for an initial condition $0 < x(0) < 1$, the slope is negative; and for an initial condition $x(0) < 0$, the slope is positive. One possible equation is

$$\dot{x} = x(x - 1).$$

Plot $\dot{x}$ versus x to verify that it actually does have the desired properties.